Evidence Relating to the Existence of the f^0 Resonance*

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A one-pion-exchange calculation indicates that the small cross section for f^0 production observed in the reaction $\pi^+p \to \pi^+\pi^-\pi^+p$ need not contradict the large values observed in the reaction $\pi^-p \to \pi^+\pi^-n$. The two types of experiment are compared for a range of incident pion momenta.

1. INTRODUCTION

THE mass distribution of $\pi^+\pi^-$ pairs produced in πp collisions has been studied in a number of recent experiments. Selove *et al.*[1] observe a peak at 1250 MeV, from the reaction

$$\pi^-p \to \pi^+\pi^-n \quad (1)$$

at 3 BeV/c, which is assigned $T=0$ and, tentatively, $J=2$ or 0. On the other hand, Steinberger *et al.*[2] apparently do not observe this peak in the $\pi^+\pi^-$ mass distribution from the related reaction

$$\pi^+p \to \pi^+\pi^-\pi^+p \quad (2)$$

at 2.62, 2.34, and 2.90 BeV/c. Since, at first sight, the experiments are quite similar, the question of a contradiction between them arises, and the identification of the peak observed in Eq. (1) with the spin-2 resonance (called the f^0) predicted from Regge-pole theory lends further interest to this. But since they are not in fact identical, one must interpret the data with the aid of theoretical assumptions. Drell[3] has suggested that the one-pion-exchange model might indicate whether the two experiments are compatible or not. The results of such a calculation, given below, indicate that they are consistent, since the f^0 peak is expected to be suppressed in experiments of the type (2).

2. ONE PION EXCHANGE CALCULATION

A. Expressions for the Cross Sections

The important one-pion-exchange diagrams for Eqs. (1) and (2) are shown in Figs. (1) and (2), respectively. [Δ^2 in Fig. 2 is the square of the four-momentum transfer to the (π^+p) isobar.] The cross section for the process of Fig. 1 is[4]

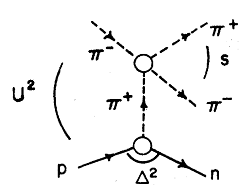

FIG. 1. Single-pion-exchange diagram for process (1).

$$\frac{\partial\sigma}{\partial s\partial\Delta^2}=\frac{1}{8\pi^2k_L^2}\sigma_{\pi\pi}\frac{[s(s-4\mu^2)]^{1/2}}{2}\frac{1}{(\Delta^2+\mu^2)^2}\frac{8\pi f^2\Delta^2}{\mu^2},$$

where k_L=incident lab momentum, $f^2=0.08$, μ is the pion mass, and $\sigma_{\pi\pi}$ is the total pion-pion cross section.[5] This includes no correction factors for off-shell effects.

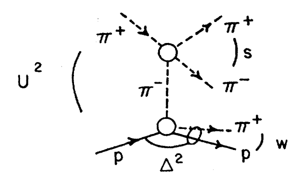

FIG. 2. Single-pion-exchange diagram for process (2).

Hence,

$$\frac{\partial\sigma}{\partial m}=\sigma_{\pi\pi}(m)\frac{m^2}{\pi k_L^2\mu^2}(m^2-4\mu^2)^{1/2}\times f^2\left\{\ln(\Delta^2+\mu^2)+\frac{\mu^2}{\Delta^2+\mu^2}\right\}_{\Delta_-^2}^{\Delta_+^2}, \quad (3)$$

where $m=\sqrt{s}$, and

$$\Delta_\pm^2=-s-\mu^2\pm[(U^2+s-W^2)^2-4U^2s]^{1/2}\times[(U^2+\mu^2-M^2)^2-4U^2\mu^2]^{1/2}/2U^2+(U^2+s-W^2)(U^2+\mu^2-M^2)/2U^2.$$

M is the nucleon mass. We have written the expression for $\Delta_\pm^2$ in a form suitable for Fig. 2; for Fig. 1 we simply put $W^2=M^2$. The cross section for the process of Fig. 2 is

$$\frac{\partial\sigma}{\partial s\partial\Delta^2\partial W^2}=\frac{1}{16\pi^3k_L^2M^2}\sigma_{\pi\pi}\sigma_{\pi N}\frac{[s(s-4\mu^2)]^{1/2}}{2}\frac{k_WW}{(\Delta^2+\mu^2)^2},$$

where k_W=momentum of π^+ associated with the recoil proton in the c.m. system of those two particles.

So, for Fig. 2,

$$\frac{\partial\sigma}{\partial m}=\frac{m^2(m^2-4\mu^2)^{1/2}}{16\pi^3k_L^2M^2}\sigma_{\pi\pi}(m)\int\int d\Delta^2dW^2\frac{\sigma_{\pi N}(W^2)k_WW}{(\Delta^2+\mu^2)^2}.$$

* Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] W. Selove, V. Hagopian, H. Brody, A. Baker, and E. Leboy, Phys. Rev. Letters **9**, 272 (1962).
[2] C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg, D. Miller, J. Schultz, J. Steinberger, T. H. Tuan, H. Brugger, P. Kramer, and R. Plano, Phys. Rev. Letters **9**, 322 (1962).
[3] S. D. Drell (private communication).
[4] F. Salzman and G. Salzman, Phys. Rev. **120**, 599 (1960).
[5] D. D. Carmony and R. T. Van de Walle, Phys. Rev. Letters **8**, 73 (1962).

TABLE I. Comparative interpretation, using the single pion exchange model, of the data of Refs. 1, 2, 7, 8, and 9. R is the expected ratio of the cross sections at the f^0 and ρ peaks.

	k_L in BeV/c	Δ_+^2 ρ	Δ_+^2 f^0	Δ_-^2 ρ	Δ_-^2 f^0	L ρ	L f^0	I ρ	I f^0	R	$\sigma_{\pi\pi}(\rho)$ (mb)
Steinberger[a]	2.9	3.43 BeV^2	1.73 BeV^2	0.094 BeV^2	0.66 BeV^2	...	...	39.47	4.35	0.07	105
Selove[b]	3	$216.7\mu^2$	$152.3\mu^2$	$0.5\mu^2$	$5.66\mu^2$	4.32	2.99	...	...	...	50
Veillet[c]	6.1	$519\mu^2$	$463\mu^2$	$0.11\mu^2$	$0.97\mu^2$	5.24	4.94	...	...	$\sim$0.6	30
Fleury[d]	10	$899\mu^2$	$844\mu^2$	$\approx$0	$0.36\mu^2$	5.79	5.69	...	...	$\sim$0.6	30
Xuong[e]	3.43	4.4 BeV^2	3 BeV^2	0.073 BeV^2	0.375 BeV^2	...	...	49.02	10.45	0.13	105

[a] See Ref. 1.
[b] See Ref. 2.
[c] See Ref. 3.
[d] See Ref. 4.
[e] See Ref. 5.

For $\sigma_{\pi N}$ we use a Breit-Wigner resonance form, so that

$$\frac{\partial\sigma}{\partial m}=\frac{m^2(m^2-4\mu^2)^{1/2}}{16\pi^3 k_L^2 M^2}\sigma_{\pi\pi}\int_0^{k_W^{\max}} dk_W\frac{2W^3}{E_f\omega}\times\left[\frac{2\pi\Gamma^2}{(\omega-\omega_0)^2+\Gamma^2/4}\right]\left[\frac{1}{\Delta^2+\mu^2}\right]_{\Delta_+^2}^{\Delta_-^2}, \quad (4)$$

where

$$W=(M^2+k_W^2)^{1/2}+(\mu^2+k_W^2)^{1/2}=E_f+\omega,$$
$$\omega_0=265\text{ MeV},\quad \Gamma=90\text{ MeV}^6$$

and

$$k_W^{\max}=[(U-m)^2-(M+\mu)^2]^{1/2}\times[(U-m)^2-(M-\mu)^2]^{1/2}/2(U-m).$$

From Eq. (1) we find the ratio of the peak heights at the f^0 and ρ masses to be

$$R=\frac{\partial\sigma}{\partial m}(m=f^0)\Big/\frac{\partial\sigma}{\partial m}(m=\rho)=\frac{f_0^2\sigma_{\pi\pi}(f_0)(f_0^2-4\mu^2)^{1/2}L(f_0)}{\rho^2\sigma_{\pi\pi}(\rho)[\rho^2-4\mu^2]^{1/2}L(\rho)}, \quad (5)$$

where

$$L(m)=\left[\ln(\Delta^2+\mu^2)+\frac{\mu^2}{\Delta^2+\mu^2}\right]_{\Delta_-^2}^{\Delta_+^2},$$

while from Eq. (2) we obtain

$$R=\frac{f_0^2(f_0^2-4\mu^2)^{1/2}\sigma_{\pi\pi}(f_0)I(f_0)}{\rho^2(\rho^2-4\mu^2)^{1/2}\sigma_{\pi\pi}(\rho)I(\rho)}, \quad (5')$$

where

$$I(m)=\int_0^{k_W^{\max}} dk_W\frac{W^3}{E_f\omega}\left(\frac{\Gamma^2}{(\omega-\omega_0)^2+\Gamma^2/4}\right)\times\left(\frac{1}{\Delta_-^2+\mu^2}-\frac{1}{\Delta_+^2+\mu^2}\right).$$

[6] J. Hamilton, *Elementary Particles* (Oxford University Press, New York, 1959), p. 322.

B. The Data of Selove and Steinberger

The experiments of Selove[1] and Steinberger[2] provide a plot of number of events/energy interval versus m. We have then $\partial\sigma/\partial m=(\delta N/N)\sigma$, where δN = number of events/energy interval, N = total number, σ = total cross section. δN and $\partial\sigma/\partial m$ are functions of m and k_L: $\delta N=\delta N(m,k_L)$, $\partial\sigma/\partial m=\partial\sigma(m,k_L)/\partial m$. For the moment, though, we are concerned only with fixed incoming momenta k_L, so we omit the second arguments. Let us write $\delta N(f^0)=R\delta N(\rho)$. We take R from Ref. 1, calculate $\sigma_{\pi\pi}(f^0)/\sigma_{\pi\pi}(\rho)$ by Eq. (5), and, hence, $\delta N(f^0)$ for Ref. 2, using Eq. (5′).

Taking $R=23/52$, we get

$$\sigma_{\pi\pi}(f^0)/\sigma_{\pi\pi}(\rho)\approx 0.13. \quad (6)$$

For Steinberger's experiment, we find

$$I(\rho)=39.47,$$
$$I(f^0)=4.35,$$

which gives $R=0.07$. Taking the peak height for ρ production as 50 events/10 MeV, we therefore estimate that one should expect between 30 and 40 events around the f^0 energy; i.e., about 3 events above background on top of a peak 100 MeV wide.

If we do not include the effect of the isobar width, we can calculate $\Delta_\pm^2$; the results are given in the first row of Table I, and are to be compared with those for Ref. 1, shown in the second row of the Table. ($1\mu^2\approx 0.02$ BeV^2.) Taking the integral over the resonant cross section to be $\frac{1}{2}\pi\Gamma\times$peak value, one then finds that the quantity corresponding to I is proportional to

$$\left(\frac{1}{\Delta_-^2+\mu^2}-\frac{1}{\Delta_+^2+\mu^2}\right)=I'.$$

Now $I'(\rho)=8.6$, and $I'(f^0)=0.9$, so that this crude approximation is quite adequate.

By way of further comparison of the model, one can calculate $\sigma_{\pi\pi}(\rho)$ from Eqs. (3) and (4); this gives

(a) $\sigma_{\pi\pi}(\rho)\approx 50$ mb from (3)

(b) $\sigma_{\pi\pi}(\rho)\approx 105$ mb from (4).

The unitarity limit is of the order of 120 mb. The result (a) is somewhat lower than the popular value of between 60 and 80 mb (Carmony and de Walle, Ref. 5) while (b) is rather higher.

C. Other Data

(1) Veillet *et al.*[7] have done the $\pi^- p$ experiment at 6.1 BeV/c. The Δ^2 values are given in the third row of Table I.

The statistics and resolution are poor (there are 60 events/100-MeV interval) but the calculated value of R is certainly consistent with their observed value of 40/60 allowing for no background. The value of $\sigma_{\pi\pi}(\rho)$ is 30 mb, very much on the low side, taking $\partial\sigma(\rho)/\partial m = 0.19$ mb/100 MeV interval.

(2) Fleury *et al.*[8] have done the same experiment at 10 BeV/c; the results of the calculation for this case are in the fourth row of Table I. In this case, the background is probably higher and it is hard to compare this estimate with the data, although an indication of a peak of the right order of magnitude is seen. From this experiment, $\sigma_{\pi\pi}(\rho)\approx$ 30 mb–40 mb.

(3) Recently, Xuong *et al.*[9] have done experiment (2) at 3.43 BeV/c. The calculated R for this case is 0.13, which is consistent with their results: They find $\sigma(\pi^+p\rightarrow\pi^+p\rho^0)=0.80\pm0.2$ mb, while the upper limit on f^0 production is 0.15 mb. These authors are continuing their analysis with a beam of higher momentum, 3.55 BeV/c. The results of the calculation are in the fifth row of the Table.

D. Discussion

The results of the calculations described in Secs. 1 and 2 are collected in Table I, in which the effects of the isobar width is neglected in the values of $\Delta_{\pm}^2$, though not in that of R. One sees that in the single-pion production experiments R is of the order $\frac{1}{2}$, whereas in the (3,3) isobar (double pion) production case R is only 0.1. This is mainly because the (πNN) vertex of Fig. 1 carries a Δ^2 in the numerator, whereas the πN scattering amplitude of Fig. 2 does not, so that the rapid variation of $(\Delta^2+\mu^2)^{-1}$ is smoothed out by the logarithm in Eq. (3). In view of the rather large values of Δ_-^2 for the experiment Eq. (2) one should also perhaps consider corrections for off-shell effects, but it is doubtful if either the accuracy of the experiments or the status of the theory warrants this.

The results are to be compared with the data of Refs. 1 and 2 and, in particular, with Fig. 3, which shows the data of Ref. 2 at the highest energy of that experiment, 2.9 BeV/c.[10] The data at the one momentum 2.9 BeV/c are the most interesting since it is only at this highest energy that the f^0 can be produced, with an isobar. They have been divided into two categories, those with $\Delta\lesssim 0.5$ BeV/c, i.e., $\Delta^2\lesssim\sim 10\mu^2$. For this Δ^2, a ρ can be made, but an f^0 cannot, with an isobar. We recall that we have not made the assumption that the isobar width is zero; and the very large value of the πN cross section at the resonance more than compensates for the smaller Δ^2 needed to make a $(\pi N f^0)$ rather than a $(N^* f^0)$ final state, so that the biggest contribution to even the f^0 production will come from the region of the N^* resonance. So we would not expect to see a peak around 1250 for the $\Delta<0.5$ BeV/c events, only for the $\Delta>0.5$ BeV/c ones. This seems to be true—there is a peak, of the right order of magnitude, around 1200 for the latter category. The reason it is shifted down in energy may be attributed to the fact that it is superposed on a sharply dropping phase space background.

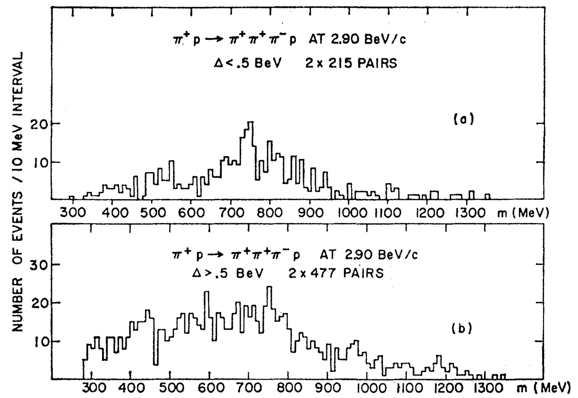

FIG. 3. Mass distribution for $\pi^+\pi^-$ pairs from $\pi^+p\rightarrow\pi^+\pi^-\pi^+p$ events (a) for $\Delta<0.5$ BeV; (b) for $\Delta>0.5$ BeV.

Our conclusion is that the data of Ref. 2 should not be taken as contradicting the evidence of Ref. 1 for the existence of the f^0 particle.

3. DEPENDENCE ON INCOMING MOMENTUM

A. Variation of the Ratio Between f^0 and ρ Production in the Reaction (2)

In view of the current experimental interest in the f^0, it is worthwhile investigating how R and $\delta N(f^0)$ vary with incoming momentum. We can relate the peak height of the f^0 peak at momentum k_L to that of the ρ peak at momentum k_{L0} by

$$\delta N(f^0,k_L)=\left[\frac{\partial\sigma}{\partial m}(\rho,k_L)\bigg/\frac{\partial\sigma}{\partial m}(\rho,k_{L0})\right]\times\frac{\delta N(f^0,k_L)}{\delta N(\rho,k_L)}\times\delta N(\rho,k_{L0}) = R'\delta N(\rho,k_{L0}). \tag{7}$$

[7] J. J. Veillet, H. Hennessy, H. Bingham, M. Bloch, D. Drijard, A. Lararrigue, P. Mittner, A. Rousset, G. Bellini, M. di Cortano, E. Fiarini, and P. Negri, Phys. Rev. Letters **10**, 29 (1963).

[8] P. Fleury, G. Kayas, F. Muller, and A. Pelletier, in *Proceedings of 1962 Annual International Conference on High-Energy Physics at CERN*, edited by J. Prentki (CERN, Geneva, 1962), p. 597.

[9] N. H. Xuong, R. Lander, and P. Yager (private communication). I am grateful to these authors for showing me their preliminary results.

[10] I am grateful to Dr. J. Schultz for these data.

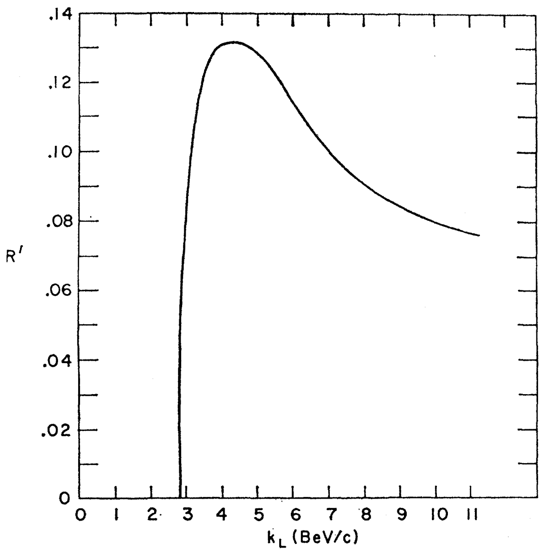

FIG. 4. Plot of R' [defined in Eq. (7)] versus k_L, the laboratory momentum of the incoming beam, with $k_{L2}=2.9$ BeV/c.

Since the first factor in R' decreases as k_L increases from k_{L0}, while the second factor—which is just R—increases, we might wonder if there is a k_L for which R' is significantly largest. It turns out, however, that the two factors in R' roughly compensate for each other: Fig. 4 shows the variation of R' with k_L, with k_{L0} the momentum involved in Ref. 2, say k_{L2}. There is only some slight advantage in doing the experiment in the range $3.5 \lesssim k_L \lesssim 5.5$ BeV/c. Beyond this upper limit the expected number of events in the f^0 peak decreases considerably. We also have

$$\frac{\partial\sigma}{\partial m}(f^0,k_L)=R'\frac{\partial\sigma}{\partial m}(\rho,k_{L2}).$$

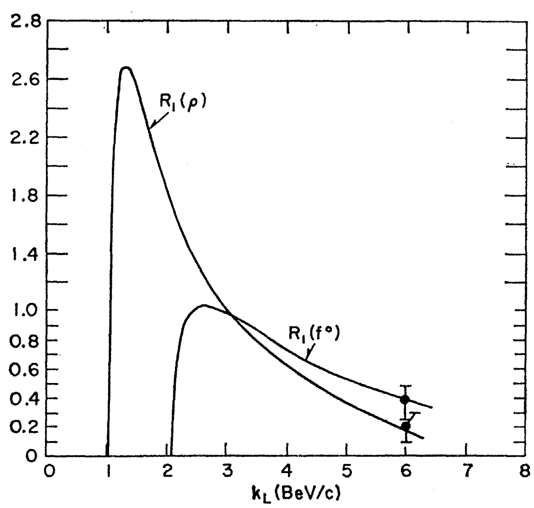

FIG. 5. The ratios of the cross sections for $\pi^-p\rightarrow\rho n[R_1(\rho)]$ and $\pi^-p\rightarrow f^0n[R_1(f^0)]$ to the corresponding cross sections of Ref. 1, as functions of the lab momentum of the incoming π^-. [See Eq. (8).] The data of Ref. 7 are shown.

Now $(\partial\sigma/\partial m)(\rho k_{L2})$ is, from Ref. 2, about 0.2 mb/10 MeV, so that considerable statistics would be needed before experiments of the type (2) gave information on the f^0 as good as those of type (1), for which the cross sections involved are about five times larger at the f^0 peak.

B. Variation of ρ and f^0 Cross Sections in the Reactions (1) and (2)

Finally, from Eq. (3) or (4) we can estimate the cross sections for ρ and f^0 production in experiments of type (1) or (2), respectively, as a function of the lab mo-

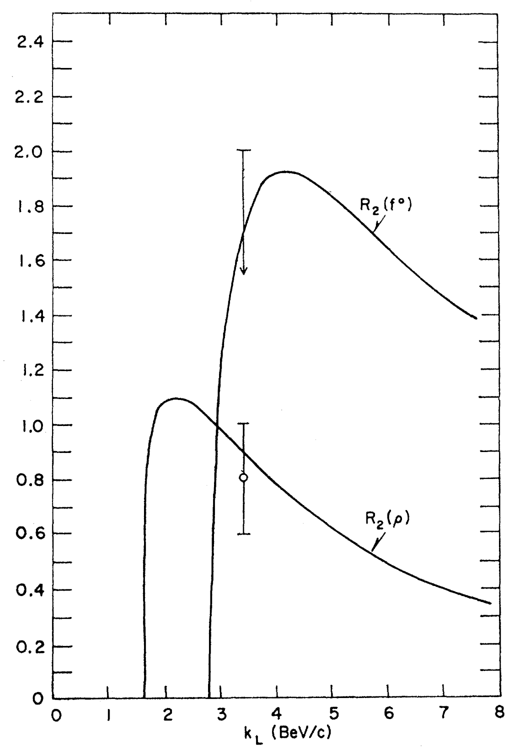

FIG. 6. The ratios of the cross sections for $\pi^+p\rightarrow\rho N^*[R_2(\rho)]$ and $\pi^+p\rightarrow f^0N^*[R_2(f^0)]$ to the corresponding cross sections of Ref. 2, as functions of the lab momentum of the incoming π^+. [See Eq. (9).] The data of Ref. 9 are shown; they provide an upper limit only in the case of $R_2(f^0)$.

mentum of the π^- or π^+. We define the ratios

$$R_1(\rho)=\frac{\partial\sigma}{\partial m}(\rho,k_L)\Big/\frac{\partial\sigma}{\partial m}(\rho,k_{L1}),$$
$$R_1(f^0)=\frac{\partial\sigma}{\partial m}(f^0,k_L)\Big/\frac{\partial\sigma}{\partial m}(f^0,k_{L1}),\tag{8}$$

and

$$R_2(\rho)=\frac{\partial\sigma}{\partial m}(\rho,k_L)\Big/\frac{\partial\sigma}{\partial m}(\rho,k_{L2}),$$
$$R_2(f^0)=\frac{\partial\sigma}{\partial m}(f^0,k_L)\Big/\frac{\partial\sigma}{\partial m}(f^0,k_{L2}),\tag{9}$$

where k_L is the (variable) incoming lab momentum, and k_{L1} and k_{L2} are the momenta of Refs. 1 and 2, respectively.[11] Then we have

$$R_1(\rho)=\frac{L(\rho,k_L)}{L(\rho,k_{L1})}\frac{k^2{}_{L1}}{k^2{}_L},\quad R_2(f^0)=\frac{I(f^0,k_L)}{I(f^0,k_{L2})}\frac{k^2{}_{L2}}{k^2{}_L}\text{ etc.}$$

The ratios R_1 and R_2 are shown in Figs. 5 and 6. We see that ρ production in experiments of type (1) is strongest below $k_L=3$ BeV/c, while f^0 production reaches its maximum there. In experiments of type (2), on the other hand, the situation is reversed: f^0 production is stronger above $k_L=3$ BeV/c, while ρ production is at its maximum there, so that, as we saw in 2(b), it is advantageous to do the experiment in the region of 4 BeV/c. As far as magnitudes are concerned, the cross sections given in Refs. 1 and 2 are, approximately,

$$\frac{\partial\sigma}{\partial m}(\rho,k_{L1})=0.52\text{ mb/50 MeV},$$

$$\frac{\partial\sigma}{\partial m}(f^0,k_{L1})=0.23\text{ mb/50 MeV},$$

$$\frac{\partial\sigma}{\partial m}(\rho,k_{L2})=0.2\text{ mb/10 MeV},$$

$$\frac{\partial\sigma}{\partial m}(f^0,k_{L2})=0.14\text{ mb/10 MeV},$$

where no correction for background has been made, and where the fourth number is based on the estimate of 1(a).

[11] R_1 and R_2 refer to reactions (1) and (2), respectively.

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Low-Energy Intranuclear Cascade Calculation*

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Monte Carlo cascade calculations have been performed for nuclear reactions involving incident protons, neutrons, π^+, and π^- on complex nuclei. The upper energy limit of validity of the calculation is $\approx$350 MeV below which pion production is not likely. In order to determine the effects of a diffuse nuclear edge, calculations were performed both for nucleon-density distributions within the nucleus which approximated the charge distribution obtained from electron-scattering data and for constant-density distributions. The results indicate that the bulk of the effect in going from a uniform to nonuniform nucleon-density distribution is due to the increased nuclear size when a diffuse edge is used, while the effects due to the diffuse edge alone are of second order. The limits of application of the general model have been investigated and are discussed.

INTRODUCTION

THE basic assumption in calculations of intranuclear cascades is that nuclear reactions involving incident particles of high energy can be described in terms of particle-particle collisions within the nucleus. The justification for this assumption is that the wavelength of the incoming particle and subsequent collision products is of the order of or smaller than the average internucleon distance within the nucleus ($\approx 10^{-13}$ cm). On the basis of this assumption, one can calculate the reaction with the nucleus by determining the life history of every particle that becomes involved in the individual particle-particle collisions occurring within the nucleus. The point of collision, the type of collision, the momentum of the struck nucleon, and the scattering angles for each collision are determined by statistical sampling techniques. Free-particle experimental data are used whenever cross-section data are required. The basic approach was suggested by Serber,[1] and statistical calculations based on his suggestion were first reported by Goldberger.[2] The latest and most complete calculation of this type was that of Metropolis *et al.*[3,4]

Some of the major features of the nuclear model used by Metropolis *et al.* are that the nucleon density within the nucleus was assumed to be a constant; a zero-

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